wave front. As the prisms were of finite size, some evidence of diffraction around the prisms was observed. Due to the tendency of the diagonal prism faces to assume a bowed position, it was impossible to obtain a gap of zero width and the gap was not of perfectly uniform width along its entire length. As the curves indicate, there is good general agreement between the experimental results and the behavior predicted theoretically. It should be noted that transmission to a receiver at location ④ occurs only in the presence of internal reflections.

Conclusion

The prism device is useable as an adjustable bidirectional coupler and as an adjustable attenuator. With coupling set at three decibels, it can be used as a hybrid junction. The present paper permits the theoretical calculation of coupling in all directions in the presence of internal reflections. The adaptability of the prism device to a wide variety of functions makes it very useful at the shorter millimeter wavelengths where the elimination of internal reflections is difficult. The calculations presented here should prove to be applicable in this case.

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Nonreciprocal Coupling with Single-Crystal Ferrites

HALVOR SKEIE

Summary—**A calculation of nonreciprocal coupling in microwave circuits with small ferrite samples tuned to ferromagnetic resonance is presented. It is shown that this coupling may be applied to the construction of simple resonant isolators, gyrators and circulators. Experimental results for the coupling in rectangular and ridge guides, applying YIG spheres, are presented. The construction of a simple *X*-band waveguide junction, acting as a 4-port resonant circulator, is described. Such a filter circulator, which may act as a switch or a frequency selective power divider, can be made tunable over the waveguide frequency range, with a bandwidth in the order of 10 Mc, and with values of insertion loss and isolation, which are comparable to those of conventional circulators.**

Introduction

In recent years, considerable attention has been given to polished ferrimagnetic single crystals for the construction of narrow band filters and microwave power limiters. This is due to their extremely low losses when used as resonating elements in filters, and the fact that coupling to microwave circuits may be achieved in a very simple way. Also, magnetic tuning may be accomplished over a wide band of frequencies.

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The author is with Laboratory of Electronics, Technical University of Norway, Trondheim, Norway.

Several constructions of band-pass and band-rejection filters have been reported,[1,2] but little attention has been paid to nonreciprocal devices, containing high Q single-crystal ferrite.[3] The band-pass filters reported by Carter[1] and others are nonreciprocal in the sense that there is a $+90°$ phase shift in one direction of propagation, and a $-90°$ phase shift in the other direction, but this nonreciprocity is not primarily wanted.

The first part of this paper consists of an analysis of the nonreciprocal coupling caused by a small single crystal sample placed in an elliptically polarized microwave magnetic field, when the sample is tuned to ferromag-

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[3] The principles of nonreciprocal coupling with application to poly-crystalline ferrite post directional couplers have been described previously in the following. R. W. Damon, "Magnetically controlled microwave directional coupler," *J. Appl. Phys.*, vol. 26, p. 1281; 1955.

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netic resonance. An extension of the previously developed equivalent circuit theory for band-pass and band-rejection filters is made, to cover nonreciprocal coupling. Formulas for the external $Q(Q_{ex})$ of the ferrimagnetic sample in waveguide structures are given.

The last part of the paper consists of a description with results of measurements on a rectangular waveguide junction with an yttrium-iron-garnet sphere used as coupling element between two guides. Tuned to ferromagnetic resonance, this junction acts as a four-port circulator.

Principle of Nonreciprocal Coupling

A small ferrite sample tuned to ferromagnetic resonance may be considered as a radiating magnetic dipole. We shall here be dealing with spherical samples only, and in that case the dipole will be circularly polarized, provided the driving field is relatively small.

If the ferrite sphere is placed in the linearly polarized magnetic field of a matched microwave transmission line, we get a reciprocal band-rejection filter which may be described by the equivalent circuit shown in Fig. 1.

The attenuation of the filter at resonance is

$$A = \frac{\text{input power}}{\text{output power}} = (1 + Q_0/Q_{ex})^2. \qquad (1)$$

The resultant fields in the transmission line may be considered as a superposition of the incident fields and the reradiation fields due to the magnetic dipole. By this, the external Q factor, Q_{ex}, may be defined as

$$Q_{ex} = \frac{\omega \cdot U_s}{P_r} \qquad (2)$$

where

ω = angular frequency
U_s = stored energy in the sphere
P_r = reradiated power from the dipole.

If the sphere is placed in an elliptically polarized magnetic field, the filter will generally be nonreciprocal. The reradiated power P_r consists of two parts, P_{rg} and P_{rl}, where

P_{rg} = reradiated power towards generator
P_{rl} = reradiated power towards load.

In the case of nonreciprocal coupling, $P_{rg} \neq P_{rl}$. From (2), we have

$$Q_{ex} = \frac{\omega \cdot U_s}{P_{rg} + P_{rl}}.$$

We may now split the coupling parameter Q_{ex} into two parts, such that.

$$1/Q_{ex} = 1/Q_{ex1} + 1/Q_{ex2} \qquad (3)$$

where

$$Q_{ex1} = \frac{\omega U_s}{P_{rg}}$$

$$Q_{ex2} = \frac{\omega U_s}{P_{rl}}.$$

The nonreciprocity of the filter is now indicated by different values of Q_{ex1} and Q_{ex2} (Fig. 2).

The attenuation formula (1) is not valid in the case of nonreciprocal coupling. We will derive a new formula by means of an expression for the incident power P_0,

$$P_0 = rr^*P_0 + P_{ab} + P_0(1 - b)(1 - b^*). \qquad (4)$$

The term rr^*P_0 denotes reflected power or power reradiated towards the generator. P_{ab} is power absorbed in the sphere, and the last term denotes transmitted power. The transmitted field is regarded as a superposition of the incident field h_0, and the reradiated field towards the load $-bh_0$, b being a coefficient of reradiation.

At the resonance point, both r and b are real, and (4) gives

$$b = \frac{2}{1 + \frac{P_{ab}}{P_0}\frac{1}{b^2} + \frac{r^2}{b^2}} = \frac{2}{1 + \frac{Q_{ex2}}{Q_0} + \frac{Q_{ex2}}{Q_{ex1}}} = \frac{2Q_l}{Q_{ex2}}. \qquad (5)$$

The unloaded Q is defined by $Q_0 = \omega U_s/P_{ab}$ and Q_l is the total loaded Q

$$1/Q_l = 1/Q_{ex1} + 1/Q_{ex2} + 1/Q_0. \qquad (6)$$

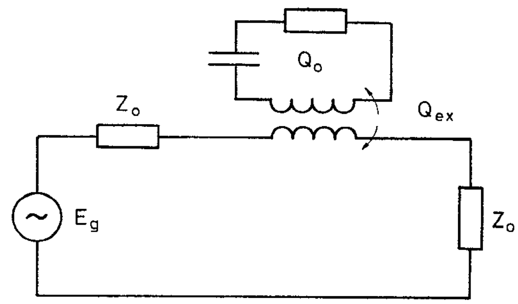

Fig. 1—Band rejection filter equivalent circuit.

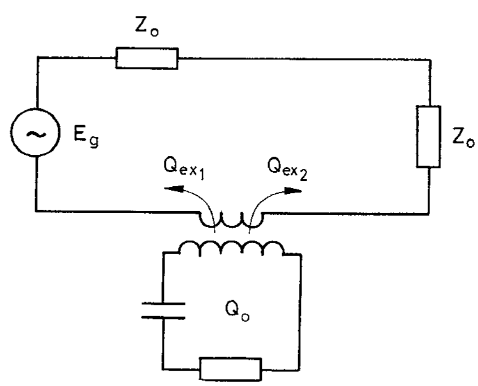

Fig. 2—Nonreciprocal filter, schematic.

Hence, the attenuation formula becomes

$$A = \frac{1}{(1-b)^2} = \frac{1}{(1 - 2Q_l/Q_{ex2})^2} \cdot \qquad (7)$$

For a reciprocal filter, $Q_{ex1} = Q_{ex2} = 2Q_{ex}$, and the formula reduces to (1).

If the sphere is placed in a circularly polarized field with coupling to a "forward" propagation of the waves, no power will be reradiated towards the generator, *i.e.*, the filter will always represent a perfect termination to the line. If a proper coupling is chosen, an ideal isolator may be obtained at the resonance point. In that case we must have

$$2Q_l/Q_{ex2} = 1$$

which means that

$$Q_{ex2} = Q_0.$$

Waves propagating in the "backward" direction through the filter will not couple to the sphere.

If the coupling is made very strong, so that $Q_l/Q_{ex2} = 1$, we see from (5) that the filter will represent a gyrator at the resonance point. We obtain 180° phase shift and practically no damping by "forward" propagation and no coupling by "backward" propagation of the waves.

General Formulas for the Coupling Parameter Q_{ex}

In the general case of nonreciprocal coupling, we assume that the ferrite sphere is placed in an elliptically polarized magnetic field in a waveguide or at the junction of several waveguides. The reradiation from the sphere at ferromagnetic resonance will generally be different in the two directions of propagation in each guide. From the definition of the external Q introduced in the preceding section, we get

$$Q_{ex1,2} = Q_0 \frac{P_{ab}}{P_{r1,2}} \qquad (8)$$

where $P_{r1,2}$ is the reradiated power in direction 1 or 2, respectively.

The absorbed power is given by

$$P_{ab} = -\operatorname{Im}\left[(1/2)\mu_0 \omega v_f \boldsymbol{m} \cdot \boldsymbol{h}^*\right] \qquad (9)$$

where

μ_0 = vacuum permeability
ω = resonance angular frequency
v_f = volume of the ferrite
$\boldsymbol{m}$ = RF magnetization of the ferrite
$\boldsymbol{h}$ = external driving RF magnetic field.

(Rationalized mks units are used, m and h measured in amperes/meter).

If it is assumed that the permanent magnetic field is directed along the z axis, we may write

$$m_x = m_0 e^{j\omega t}, \qquad m_y = -jm_0 e^{j\omega t}.$$

The RF magnetization amplitude m_0 is related to the external RF magnetic field amplitudes by

$$m_0 = \chi_{xx}{}^e h_{x0} + \chi_{xy}{}^e h_{y0},$$

where $\chi_{xx}{}^e$, $\chi_{xy}{}^e$ are components of effective (external) RF tensor susceptibility.

An approximate solution of the equation of motion for the magnetization vector yields at the resonance point (see Carter[1])

$$\chi_{xx}{}^e = -j\frac{Q_0\omega_m}{\omega}, \qquad \chi_{xy}{}^e = \frac{Q_0\omega_m}{\omega}$$

where $\omega_m = |\mu_0 \gamma M_0|$, γ being the gyromagnetic ratio, and M_0 the saturation magnetization of the ferrite. By this is obtained

$$m_0 = -j\frac{Q_0\omega_m}{\omega}(h_{x0} + jh_{y0}). \qquad (10a)$$

Assuming an elliptically polarized field such that

$$h_y = -j\epsilon h_x,$$

where ϵ is a real factor, we may write

$$m_0 = -j\frac{Q_0\omega_m}{\omega} h_{x0}(1+\epsilon). \qquad (10b)$$

Inserting this into (9), we obtain

$$P_{ab} = 1/2\mu_0\omega^2 v_f \frac{m_0^2}{Q_0\omega_m} \cdot \qquad (11)$$

From the assumption that the reradiation fields couple to the sphere in the same way as the originally impressed field, we obtain, for coupling to m_x,

$$P_{rx} = 1/2\mu_0\omega v_f m_0 h_{rxx}, \qquad (12a)$$

for coupling to m_y,

$$P_{ry} = 1/2\mu_0\omega v_f m_0 h_{ryy}. \qquad (12b)$$

h_{rxx} denotes the x-component amplitude of the reradiation field due to m_x, and h_{ryy} denotes the y-component amplitude of the reradiation field due to m_y. We will denote the direction of propagation along the negative y axis by the suffix 1, and the direction along the positive y axis by the suffix 2. The power of an incident wave along direction 2 may be expressed as

$$P_{in} = K_1 h_{0x}^2 = K_2 h_{0y}^2 \qquad (13)$$

where h_{0x} and h_{0y} are the magnetic field components (amplitudes) at the position of the sphere without the ferrite. K_1 and K_2 are constants given by the coupling structure and the position of the sphere.

The reradiation field amplitude h_{rxx} and h_{ryy} may then be found from

$$K_1 h_{rxx}^2 = 1/4\mu_0\omega v_f m_0 h_{rxx}$$
$$K_2 h_{ryy}^2 = 1/4\mu_0\omega v_f m_0 h_{ryy} \qquad (14)$$

which leads to

$$h_{rxx} = \pm \frac{\mu_0 \omega v_f m_0}{4K_1}$$

$$h_{ryy} = \pm \frac{\mu_0 \omega v_f m_0}{4K_2} \cdot \quad (15)$$

If $h_y = -j\epsilon h_x$, we get for the total reradiation field along direction 2

$$h_{rx2} = h_{rxx} + h_{ryx} = \pm \frac{\mu_0 \omega v_f m_0}{4K_1}(1+\epsilon). \quad (16a)$$

If h_{0x} is assumed to have a symmetric x dependence and h_{0y} an antisymmetric x dependence, we similarly get for the total reradiation field along direction 1,

$$h_{rx1} = \pm \frac{\mu_0 \omega v_f m_0}{4K_1}(1-\epsilon). \quad (16b)$$

By insertion into (8) we finally obtain

$$Q_{ex1} = \frac{8K_1}{\mu_0 \omega_m v_f} \frac{1}{(1-\epsilon)^2} \quad (17a)$$

$$Q_{ex2} = \frac{8K_1}{\mu_0 \omega_m v_f} \frac{1}{(1+\epsilon)^2} \cdot \quad (17b)$$

When $\epsilon = 0$ or $\epsilon \pm \infty$, the coupling is reciprocal, and

$$Q_{ex1} = Q_{ex2} = \frac{8K_1}{\mu_0 \omega_m v_f} \quad \text{for } \epsilon = 0$$

$$Q_{ex1} = Q_{ex2} = \frac{8K_2}{\mu_0 \omega_m v_f} \quad \text{for } \epsilon \to \pm \infty.$$

When $\epsilon = \pm 1$, the magnetic RF fields are circularly polarized, and either Q_{ex1} or Q_{ex2} will be infinite.

Analysis and Measurements of Coupling Parameters in Rectangular and Ridge Waveguides

In a matched rectangular wave guide, propagating power in its principal mode, the magnetic field is elliptically polarized. The ellipticity and the direction of rotation vary as a function of the coordinate x, the field being linearly polarized at the center, and at the side walls of the guide, *i.e.*, for $x=0$ and $x = \pm a/2$ (Fig. 3).

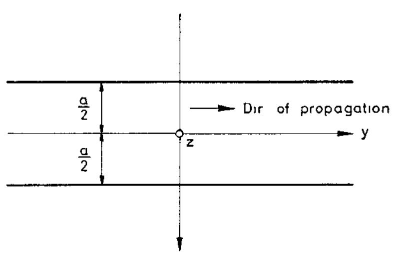

Fig. 3—Diagram showing position of coordinate system and direction of propagation chosen for the rectangular waveguide.

The RF magnetic field in the guide is given by

$$h_x = h_w \frac{\lambda_c}{\lambda_g} \cos \frac{\pi x}{a}$$

$$h_y = j h_w \sin \frac{\pi x}{a} \cdot$$

Here, h_w denotes the magnetic field at the side walls; λ_c is the cutoff wavelength, and λ_g is the propagation wavelength in the guide. The incident power is given by

$$P_{\text{in}} = 1/4 \frac{\lambda_c^2}{\lambda \cdot \lambda_g} z_0 a b h_{w0}^2,$$

where λ denotes free space propagation wavelength, z_0 denotes the impedance of free space, b is the height of the guide, and h_{w0} is the amplitude of the magnetic field at the side walls. With an arbitrary position of the ferrite sphere, this yields for the constants K_1 and K_2 (13)

$$K_1 = 1/4 \frac{\lambda_g}{\lambda} z_0 a b \frac{1}{\cos^2 \frac{\pi x}{a}} \quad (18a)$$

$$K_2 = 1/4 \frac{\lambda_c^2}{\lambda \lambda_g} z_0 a b \frac{1}{\sin^2 \frac{\pi x}{a}} \quad (18b)$$

with $\epsilon = h_y/jh_x = \lambda_g/\lambda_c \tan \pi x/a$, and putting $\lambda_c/\lambda_g = \tan \alpha$, we obtain

$$Q_{ex1} = \frac{2z_0 ab}{\mu_0 \omega_m v_f} \frac{\lambda_g}{\lambda} \frac{\sin^2 \alpha}{\sin^2\left(\alpha - \frac{\pi x}{a}\right)} \quad (19a)$$

$$Q_{ex2} = \frac{2z_0 ab}{\mu_0 \omega_m v_f} \frac{\lambda_g}{\lambda} \frac{\sin^2 \alpha}{\sin^2\left(\alpha + \frac{\pi x}{a}\right)} \cdot \quad (19b)$$

The coupling is reciprocal when the sphere is placed at the center of the guide, or at the side walls. At the center

$$Q_{ex1} = Q_{ex2} = \frac{2z_0 ab}{\mu_0 \omega_m v_f} \frac{\lambda_g}{\lambda} \cdot$$

At the side walls

$$Q_{ex1} = Q_{ex2} = \frac{2z_0 ab}{\mu_0 \omega_m v_f} \frac{\lambda_c^2}{\lambda \lambda_g} \cdot$$

When $\lambda_g = \lambda_c$, the coupling is equal at the center and at the side walls, and the attenuation formula (7) takes the form

$$A = \left[\frac{1 + \frac{Q_0 \mu_0 \omega_m v_f}{z_0 ab} \frac{\lambda}{\lambda_g}}{1 - \frac{Q_0 \mu_0 \omega_m v_f}{z_0 ab} \frac{\lambda}{\lambda_g} \sin\left(\frac{2\pi x}{a}\right)} \right]^2 \cdot \quad (20)$$

Measured values of the attenuation at resonance for an X-band waveguide at $f=9000$ Mc, applying a 0.02 inch diameter YIG sphere and at $f=9250$ Mc applying a 0.077 inch diameter YIG sphere are plotted in Figs. 4 and 5.

The dashed curves are theoretical curves calculated from (20). Theoretical and experimental curves are made to coincide at the point of circularly polarized fields, which corresponds to a linewidth of 0.94 oersted for the small sphere and a linewidth of 0.41 oersted for the larger sphere. Some magnetostatic mode coupling was present in the measurements on the largest sphere. The small sphere provides a weak coupling and no points of infinite damping are present. The larger sphere provides a very strong coupling with only 1.7 db damping at the point of circularly polarized fields (gyrator filter) and two points of infinite damping, one close to the center and one close to the side wall of the guide. If an ideal isolator (uniline) should be wanted at 9000 Mc, a sphere size somewhere between these two should have to be chosen.

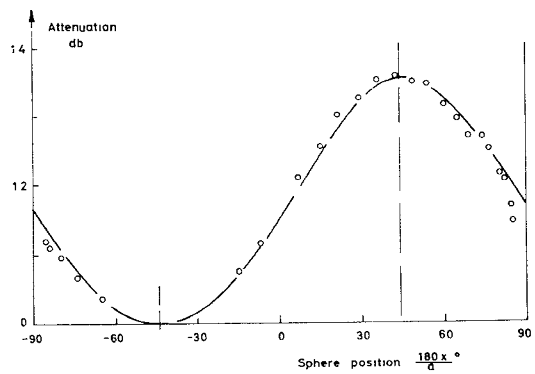

Fig. 4—Attenuation at resonance as a function of sample position for a 0.02 inch sphere placed in a rectangular waveguide. Frequency $f=9000$ Mc.

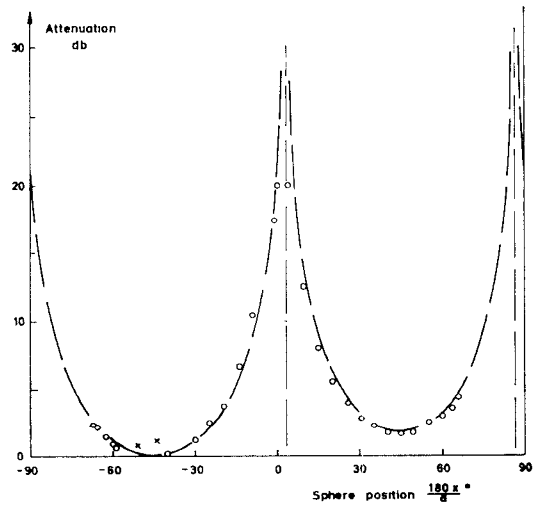

Fig. 5—Attenuation at resonance as a function of sample position for a 0.077 inch sphere, placed in a rectangular waveguide. Frequency $f=9250$ Mc.

It is obvious that similar nonreciprocal band-rejection filters can be obtained by application of ridge guides. The field configuration is more complicated and more difficult to compute accurately than for ordinary rectangular guides, but a useful method of approximate field calculation is described by Getzinger.[4] With notations as shown in Fig. 6, he has evaluated

$$h_x = -\frac{z_0 k}{k_y}\frac{H_x}{E_0} = \frac{d\cos k_c s/2}{b\sin k_c\cdot l}\sin k_c s + \sum_{n=1}^{\infty}\frac{2}{n\pi}\frac{\cos k_c s/2}{\sin h\gamma_n l}\cdot\sin\frac{n\pi d}{b}\sinh\gamma_n x\cos\frac{n\pi d}{b}$$

$$h_y = -\frac{jz_0 k H_y}{k_c E_0} = \cos k_c s/2\left\{\frac{d\cos k_c x}{b\sin k_c l} - \sum_{n=1}^{\infty}\frac{2k_c\sin n\pi d/b}{n\pi\gamma_n\sinh\gamma_n l}\cosh\gamma_n x\cos\frac{n\pi z}{b}\right\} \tag{21}$$

in which

z_0 = free space impedance
k = free space wave number
k_y = waveguide wave number along y
k_c = free space wave number at the cutoff frequency
H_x = actual x-component of the magnetic field
H_y = actual y-component of the magnetic field
E_0 = electric field at the center, above the ridge

$$\gamma_n = \sqrt{\left(\frac{n\pi}{b}\right)^2 - k_c^2}.$$

Eq. (21) gives a solution for H_x and H_y in terms of E_0. The incident power is

$$P_{\text{in}} = \frac{1}{2Z_{pv}}(E_0 d)^2. \tag{22}$$

Z_{pv} is the power-voltage impedance and may be calculated from a formula given by Mihran.[5]

To investigate this coupling, values of the field components h_x and h_y were computed for several values of x and z for two different ridge guides on the electronic computer GIER. By use of these results [and (21)] "theoretical" attenuation curves for several frequencies were calculated. Some measurements were also made. The results from the measurements on a ridge guide with dimensions,

$$a = 22.7 \text{ mm}$$
$$b = 10.0 \text{ mm}$$
$$l = 8.6 \text{ mm}$$
$$d = 1.5 \text{ mm},$$

[4] W. J. Getzinger, "Ridge waveguide field description and application to directional couplers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-1, pp. 41–50; January, 1962.
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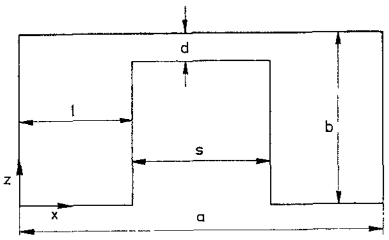

Fig. 6—Ridge guide cross section.

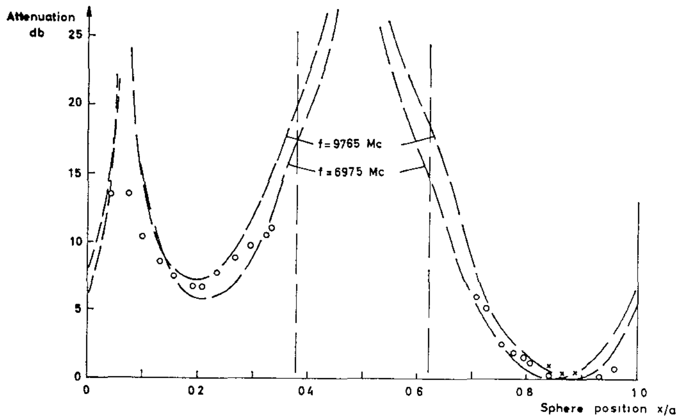

Fig. 7—Attenuation at resonance as a function of sphere position for a single ridge guide at the frequency $f=9250$ Mc. Theoretical curves for $f=6975$ Mc and $f=9765$ Mc are dashed.

are plotted in Fig. 7. The 0.077 inch YIG sphere was used at the frequency of 9250 Mc. Theoretical curves for 6975 Mc and 9765 Mc, based on a linewidth of 0.43 oersted, are shown as dashed lines for comparison.

Development of a Tunable Circulator

A nonreciprocal waveguide junction is easily obtained by means of ferromagnetic resonance coupling. Consider the junction shown in Fig. 8. Two rectangular waveguides are separated by a thin wall at the junction. The ferrite sphere is placed in a small hole in the wall at the coordinates x, y. With matched terminations at all 4 arms, the equivalent circuit shown in Fig. 9 may describe the coupling at ferromagnetic resonance. The junction may be described by its scattering matrix

$$S = \begin{pmatrix} S_{11} \cdots S_{14} \\ S_{21} \cdots S_{24} \\ S_{31} \cdots S_{34} \\ S_{41} \cdots S_{44} \end{pmatrix}.$$

If a wave of power P_0 is incident at port 1, the power equation yields

$$P_0 = P_0 |S_{11}|^2 + P_0 |S_{12}|^2 + P_0 |S_{13}|^2 + P_0 |S_{14}|^2 + P_0 a^2, \tag{23}$$

where $P_0 a^2$ is the power absorbed by the ferrite.

According to Figs. 8 and 9 we get

$$\left|\frac{S_{11}}{a}\right|^2 = \frac{Q_0}{Q_{ex1}}, \quad \left|\frac{S_{12}}{a}\right|^2 = \frac{Q_0}{Q_{ex2}}$$

$$\left|\frac{S_{13}}{a}\right|^2 = \frac{Q_0}{Q_{ex3}}, \quad \left|\frac{S_{14}}{a}\right| = \frac{1}{a} - \sqrt{\frac{Q_0}{Q_{ex4}}}. \tag{24}$$

From (23) and (24) and by a similar reasoning for power incident at the other ports, we obtain the total scattering matrix (the ferrite sample is the reference point)

$$S = \begin{pmatrix} \pm \frac{2Q_l}{\sqrt{Q_{ex1}Q_{ex4}}}, & \mp j \frac{2Q_l}{\sqrt{Q_{ex2}Q_{ex4}}}, & \pm j \frac{2Q_l}{\sqrt{Q_{ex3}Q_{ex4}}}, & 1 - \frac{2Q_l}{Q_{ex4}} \\ \pm j \frac{2Q_l}{\sqrt{Q_{ex1}Q_{ex3}}}, & \pm \frac{2Q_l}{\sqrt{Q_{ex2}Q_{ex3}}}, & 1 - \frac{2Q_l}{Q_{ex3}}, & \mp j \frac{2Q_l}{\sqrt{Q_{ex3}Q_{ex4}}} \\ \mp j \frac{2Q_l}{\sqrt{Q_{ex1}Q_{ex2}}}, & 1 - \frac{2Q_l}{Q_{ex2}}, & \pm \frac{2Q_l}{\sqrt{Q_{ex2}Q_{ex3}}}, & \pm j \frac{2Q_l}{\sqrt{Q_{ex2}Q_{ex4}}} \\ 1 - \frac{2Q_l}{Q_{ex1}}, & \pm j \frac{2Q_l}{\sqrt{Q_{ex1}Q_{ex2}}}, & \mp j \frac{2Q_l}{\sqrt{Q_{ex1}Q_{ex3}}}, & \pm \frac{2Q_l}{\sqrt{Q_{ex1}Q_{ex4}}} \end{pmatrix}.$$

Q_l is the total loaded Q. (The sign $\pm$ depends on the position of the sphere.)

If the sphere is placed at the point of circularly polarized fields for both guides ($\pi x/a = \pi y/a = \alpha$), a resonant circulator is obtained. The scattering matrix will be

$$S = \begin{pmatrix} 0, & -j \frac{2Q_l}{\sqrt{Q_{ex2}Q_{ex4}}}, & 0, & 1 - \frac{2Q_l}{Q_{ex4}} \\ 0, & 0, & 1, & 0 \\ 0, & 1 - \frac{2Q_l}{Q_{ex2}}, & 0, & j \frac{2Q_l}{\sqrt{Q_{ex2}Q_{ex4}}} \\ 1, & 0, & 0, & 0 \end{pmatrix}.$$

Since the field[6] at the center of a small hole in an infinitely thin wall must be half the field at the wall outside the hole, the Q-values are readily found to be

$$Q_{\text{ex}1} = 4Q_{\text{ex}0}\frac{\sin^2\alpha}{\sin^2\left(\alpha - \frac{\pi y}{a}\right)}$$

$$Q_{\text{ex}2} = 4Q_{\text{ex}0}\frac{\sin^2\alpha}{\sin^2\left(\alpha + \frac{\pi x}{a}\right)}$$

$$Q_{\text{ex}3} = 4Q_{\text{ex}0}\frac{\sin^2\alpha}{\sin^2\left(\alpha - \frac{\pi x}{a}\right)}$$

$$Q_{\text{ex}4} = 4Q_{\text{ex}0}\frac{\sin^2\alpha}{\sin^2\left(\alpha + \frac{\pi y}{a}\right)} \quad (25)$$

where

$$Q_{\text{ex}0} = \frac{2z_0ab}{\mu_0\omega_m v_f}\frac{\lambda_g}{\lambda},$$

when it is assumed that the two guides have equal dimensions.

An experimental circulator of this kind was constructed by use of X-band waveguides and a 0.077 inch diameter YIG sphere. The waveguide junction construction is shown in Fig. 10. The insertion wall is 0.1 mm thick and the hole has a diameter of 3.5 mm. The hole is placed half way between the center and the side wall of each guide, which yields circularly polarized fields for $\alpha = 45°$, or $\lambda_s = \lambda_c$, *i.e.*, at approximately 9250 Mc. The theoretical scattering matrix, based on an unloaded Q of the sphere of 8100 was calculated for 9250 Mc.

$$S_{\text{theoret}} = \begin{bmatrix} 0, & -j0.835 & 0, & 0.165 \\ 0, & 0 & 1, & 0 \\ 0, & 0.165, & 0, & j0.835 \\ 1, & 0, & 0, & 0 \end{bmatrix}.$$

Coupling through the hole without ferrite is neglected.

The following scattering matrix was measured.

$$S_{\text{exp}} = \begin{bmatrix} 0.05, & -j0.84, & -j0.08, & 0.156 \\ (j0.06) & (-0.025) & (0.96) & (0.07) \\ j0.08 & 0.18 & 0.05 & j0.835 \\ (0.96) & (-j0.05) & (-j0.05) & (-0.025) \end{bmatrix}.$$

[6] H. A. Bethe, "Theory of diffraction by small holes," *Phys. Rev.*, vol. 66, nos. 7 and 8, pp. 163–173; Oct. 1 and 15, 1944.

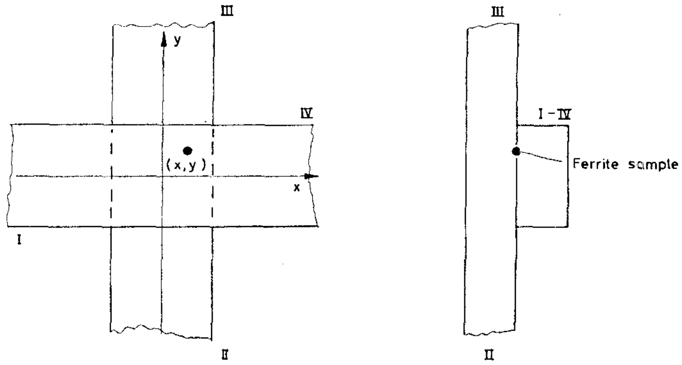

Fig. 8—Nonreciprocal rectangular waveguide junction.

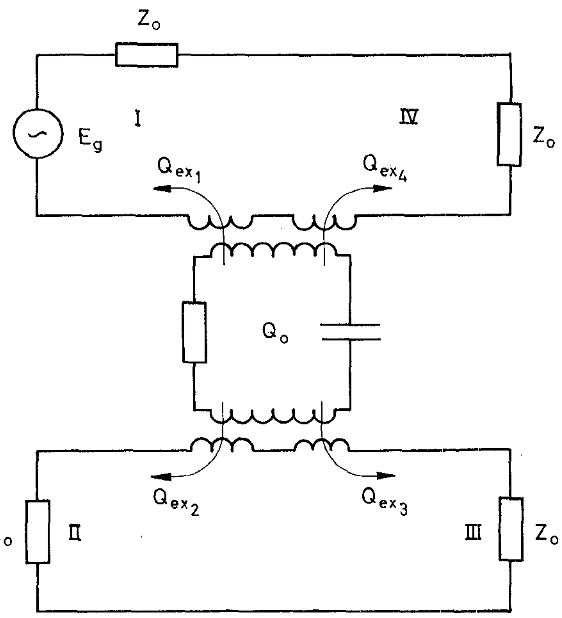

Fig. 9—Four-arm waveguide junction, equivalent circuit.

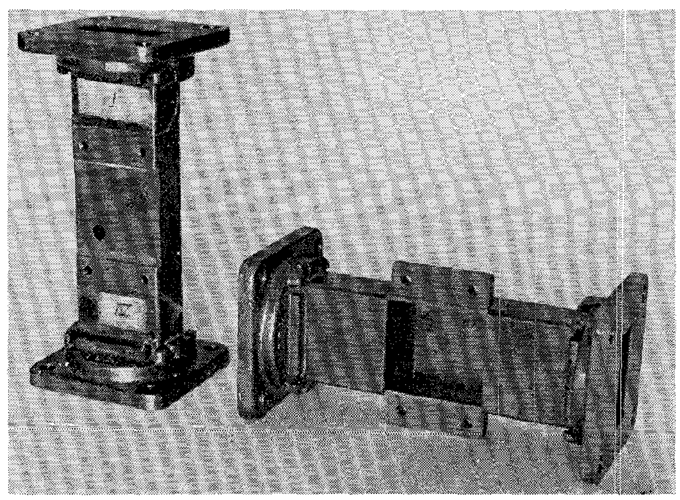

Fig. 10—Photograph showing the experimental waveguide junction construction (disassembled).

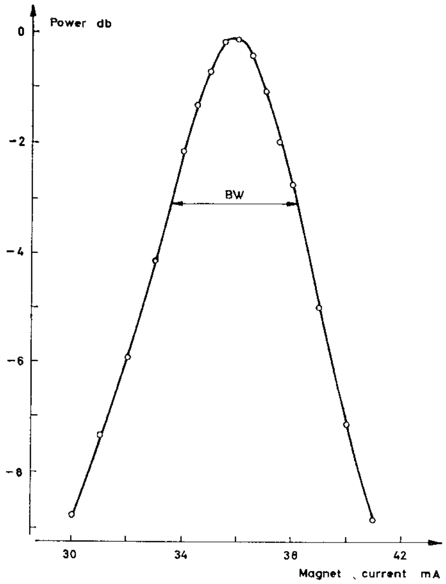

Fig. 11—Transmitted power from port I to port II as a function of magnet current at 9250 Mc.

We see that the sphere of 0.077 inch diameter is not large enough to get better isolation than 16–17 db, for transmission 1–2. The best way to get large enough coupling to increase this value considerably, and at the same time avoid too much magnetostatic mode coupling and coupling outside resonance, would probably be to apply two spheres, one on each side of the intersection wall. The band-pass curve should then get the form of a typical two-resonator response curve. A reduction of the guides height should also be preferred to get larger coupling and to minimize the weight and dimensions of the dc magnet.

Conclusions

Nonmechanically tuned, narrow band filters are of considerable interest for application in microwave circuits. The use of nonreciprocal coupling should contribute to expand the range of applicability of these filters. On the basis of the coupling structures described in this paper it is possible to construct

TABLE I

Theoretical and Experimental Reflection and Transmission Coefficients of an X-Band Waveguide Junction at 8000 Mc.

	Theoretical Values	Measured Values
Reflection S_{11}	$-$ 0.148 or VSWR 1.35	VSWR 1.22
Transmission S_{12}	$-j$0.835 or damping 1.6 db	damping 1.9 db
Transmission S_{13}	$-j$0.148 or damping 16.6 db	damping 17.7 db
Transmission S_{14}	0.165 or damping 15.7 db	damping 17.9 db
Reflection S_{22}	$-$ 0.148 or VSWR 1.35	VSWR 1.27
Transmission S_{21}	j0.026 or damping 31.7 db	damping 21.7 db
Transmission S_{23}	0.974 or damping 0.23 db	damping 0.2 db
Transmission S_{24}	j0.148 or damping 16.6 db	damping 18.5 db

The values within the brackets have probably been influenced by magnetostatic mode coupling, which was present to some extent (about 20 db weaker than the main resonance).

The isolation outside resonance was measured and found to be 28–35 db. The 3-db bandwidth for transmission 1–2 was measured as 7.2 Mc, which agrees very well with the theoretical value. The band-pass curve for transmission 1–2 is shown in Fig. 11.

To investigate how the junction could work as a tunable circulator filter, it was tuned to ferromagnetic resonance at 8000 Mc and measurements of transmission coefficients were made. A calculation of some transmission coefficients was also made on the basis of the theoretical formulas. Theoretical and experimental values are listed in Table I. A somewhat larger magnetostatic mode coupling occurred at this frequency.

The calculated and experimental results show that the circulator should operate fairly well within the tuning range 8000–11,000 Mc.

1) Stop-band filters with no reflection at the input (resonant isolators).
2) Frequency dependent phase shifters (gyrator filters).
3) 4-port resonant circulators which also can be used as frequency selective switches or dividers. (It is possible to pick out the signal at a particular frequency or a narrow band of frequencies, leaving the signals present at other frequencies undisturbed in the input guide and with no reflection of the separated signal.)
4) Reflection-free resonant power limiters.

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